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Comparative Study of Nonreflecting Boundary Condition for One-Step Duct Aeroacoustics Simulation

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Introduction

ONE-STEP numerical simulations of aeroacoustics problems have been studied for quite some time. The proposed methods usually solve the fully unsteady compressible Navier–Stokes equations, thus allowing the far-field sound and the near-field aerodynamics to be determined without modeling the source terms in the wave equation. Because of the very small energy of the acoustics field, a low dispersive and low dissipative scheme is always required if wave propagation were to be resolved accurately in an aeroacoustics computation. Besides, precise boundary conditions also play a key role in aeroacoustics computations. At the inflow and outflow boundaries, the assumed computational boundaries should allow the aerodynamic field to pass freely with minimal reflection, while at the same time they should be nonreflecting for the incident acoustic waves. Otherwise the spurious erroneous waves reflecting from the boundaries would contaminate the numerical simulation, decrease the computational accuracy, and might even drive the solution toward a wrong time-stationary state. Therefore, it is necessary to formulate truly nonreflecting conditions at these computational boundaries.

The most widely used Navier–Stokes characteristics-based boundary conditions (NSCBC) (see Refs. 1–4) for unsteady flows has been proven accurate only if the wave incidence is normal to the computational boundary. Numerical instabilities at other incident angles are intolerable; the instabilities are further amplified in the presence of strong mean shear. Consequently, damping techniques or filtering with a buffer region between the physical domain and outflow boundaries is commonly invoked in the use of NSCBC.[5] In addition to suppressing instabilities, the damping or filtering is so constructed that it renders the flow at the end of the buffer region more one dimensional; thus, the residual wave will be relatively more normal to the computational boundary prescribed for NSCBC.

Two kinds of buffer regions are commonly adopted for one-step aeroacoustics simulation: the absorbing boundary condition (ABC) proposed by Freund[6] and the method of perfectly matched layer (PML) proposed by Hu.[7] In the Freund ABC, damping terms are added to the original Navier–Stokes equations and are activated only in the buffer region. The damping terms are usually multiplied by an absorption coefficient σ, which is defined as $\sigma = \sigma_m(\delta/D)^2$, where σ_m is a constant to be specified, δ is the distance measured from the start of the damping region, and D is its width. Thus defined, there is zero damping at the entrance to the damping region, and the damping terms would eliminate all disturbances, aerodynamic and acoustic, and drive the solution to a desired outlet flowfield, which is known a priori. In PML, the damping terms are constructed based on a linear analysis of the corresponding linearized Euler problem. Absorption coefficients similar to those used in the Freund ABC scheme is also assumed. Thus formulated, the PML is able to smoothly absorb acoustic, vorticity, and entropy disturbances. Both nonreflecting schemes work satisfactorily in aeroacoustics simulation in an infinite medium. To prevent any leakage of disturbances into the physical domain, the size of the buffer region is usually made quite large. If the simulated problem is more complex, for example, the presence of a solid boundary, it is found that the size of the buffer region could be comparable to the physical domain.[8,9] It is expected that the difficulty could be further compounded if acoustic propagation occurs inside a duct with discontinuities.

One of the difficulties could be attributed to vorticity disturbances that are created at geometrical discontinuities.[10] These disturbances might create additional acoustic waves, which might undergo multiple reflections, and then interact with convecting vorticities. Eventually the directions, phases, and amplitudes of the acoustic and vortical waves in the inflow and outflow would vary significantly and gave rise to critical problems at the computation boundary. This Technical Note attempts to compare and examine the suitability of the aforementioned nonreflecting boundary conditions for one-step simulation of duct aeroacoustics. Only the Freund ABC[6] and the PML proposed by Hu[7] are compared. In this Note, the accuracy and the size of the computational domains are considered in the comparison. A reference solution is used as benchmark. The reference solution is obtained by using the Freund ABC[6] with larger physical and computational domains.

Numerical Solution of the Governing Equations

The governing equations are the unsteady, compressible Navier–Stokes equations written in strong conversation form in two dimensions. These equations can be made dimensionless by normalizing the variables using reference quantities such as U_{ref}, ρ_{ref}, T_{ref}, L_{ref}, and $L_{\mathrm{ref}}/U_{\mathrm{ref}}$. A direct numerical simulation (DNS) method is used to solve the nondimensional governing equations. They are solved by a five-point, sixth-order compact finite difference scheme suggested by Lele[11] to obtain the spatial derivative and an explicit fourth-order Runge–Kutta scheme for time marching. The high-order filtering derived by Visbal and Gaitonde[12] is applied in every final stage of the Runge–Kutta scheme to suppress numerical instabilities due to spatial differencing. At all computational boundaries, NSCBC is applied to close effectively the Navier–Stokes equations.

Altogether, two cases are calculated: case 1 is a uniform subsonic flow over a cavity, and case 2 deals with acoustic wave propagation past a backward-facing step expansion in a duct. Case 1 gives aeroacoustics interactions in a semi-infinite plane, whereas case 2 yields the full complexity of duct aeroacoustics with geometric discontinuites. To compare the performance of the Freund ABC and the

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Table 1 Physical and computational domains for two cases studied[a]

Computational region	Buffer region width[b]	Grid stretching in buffer region, Yes/No
	Case 1[c], nonuniform grid, $\Delta x_{min} = \Delta y_{min} = 0.001$	
	Freund ABC (reference solution)	
x: $-6.4 \le x \le 13.6$ y: $-1 \le y \le 9.9$	$D_I = 2$, $D_T = 1.9$, and $D_O = 7.7$	No
	Freund ABC	
x: $-2.6 \le x \le 5.8$ y: $-1 \le y \le 3.9$	$D_I = 1.6$, $D_T = 1.9$, and $D_O = 3.1$	No
	PML	
x: $-2.6 \le x \le 5.8$ y: $-1 \le y \le 3.9$	$D_I = 1.6$, $D_T = 1.9$, and $D_O = 3.1$	No
	Case 2[d], nonuniform grid, $\Delta x_{min} = 0.05$ and uniform grid, $\Delta y = 0.05$	
	Freund ABC (reference solution)	
x: $-4 \le x \le 100$ y: $-1 \le y \le 1$	$D_I = 1$ and $D_O = 37.3$	No
	Freund ABC	
x: $-4 \le x \le 60$ y: $-1 \le y \le 1$	$D_I = 1$ and $D_O = 31.2$	No
	PML	
x: $-4 \le x \le 60$ y: $-1 \le y \le 1$	$D_I = 1$ and $D_O = 31.2$	No

[a]All lengths are normalized by reference lengths of respective cases.
[b]D_I, inlet; D_O, outlet; and D_T, top.
[c]Flow over cavity with $M = 0.6$ and $Re_H = 1499.5$.
[d]Acoustic wave propagation over backstep expansion in duct with $M = 0.1$ and $Re_H = 400$.

PML, the same absorption coefficient, that is, $\sigma = \sigma_m(\delta/D)^2$, is assumed for the damping terms in the calculations of these two cases. Furthermore, σ_m is taken to be 20 and D is as defined in Table 1, where a comparison of the physical and computational domains, grid size, and buffer zone width is listed. The use of grid stretching or lack thereof for the Freund ABC and PML is also listed in Table 1. The isothermal no-slip condition[3] is applied to all solid boundaries. The deviation of any calculated variable h from the reference solution (denoted with subscript r) is measured in terms of integral error norms:

$$L_2(h) = \sqrt{\sum_{k=1}^{N}(h_k - h_{k,r})^2 \Big/ N}$$

and $L_\infty(h) = \max_k |h_k - h_{k,r}|$, where N is the total number of grid points and k is just an index. Unless otherwise specified, all variables in the following discussion are dimensionless.

Results and Discussion

Subsonic Flow over a Cavity

Referring to Fig. 1, the test problem has the following specifications: L_{ref} is taken to be the cavity depth H, $U_{\text{ref}} = U_\infty$, the cavity length is $L = 2H$, and the dimensionless flow parameters are $U_\infty = 1$, $V_\infty = 0$, $\rho_\infty = 1$, $T_\infty = 1$, $M = 0.6$, $\theta_0 = 0.03788$, $Re_{\text{H}} = \rho_\infty U_\infty H/\mu = 1499.5$, and $Re_\theta = \rho_\infty U_\infty \theta_0/\mu = 56.8$, where θ_0 is the initial momentum thickness at the leading edge of the cavity. These parameters are identical to the 2M6 case ($L/H = 2$, $M = 0.6$) by Rowley et al.[8] The initial condition is a laminar flat-plate boundary layer along the wall. An incompressible Blasius boundary-layer profile is used to initialize the velocity distribution in the boundary layer. A nonuniform Cartesian grid, which is highly clustered near the wall, is used to carry out the simulation.

Both the Freund ABC and PML schemes require a prescribed far-field solution at the boundaries of the buffer region. Usually, a far-field uniform flow is specified. This specification could be approximately satisfied if the physical and computational domains are very large. Even if the uniform flow requirement is satisfied, there is also the question of what to assume for the flow inside the boundary layer, which is far from uniform. Therefore, the boundary layer would need special treatment if this aeroacoustics problem were to be simulated correctly. The far-field solution should be as similar to the mean boundary layer of the problem as possible; otherwise, the deviation of the far-field solution from the actual boundary physics would create fallacious flow instability and more error for the calculations. In this study, an incompressible Blasius boundary-layer profile is assumed.

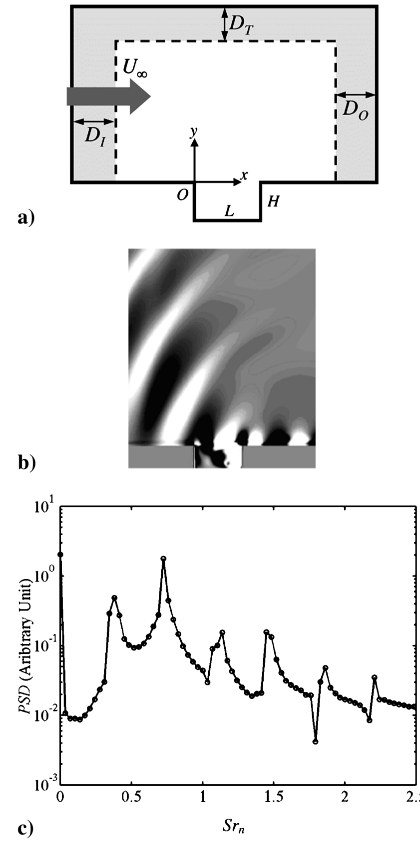

Fig. 1 Flow past a cavity with $M = 0.6$: a) computational domain and buffer regions, b) profiles of density gradient $\partial\rho/\partial x$ from the calculations, and c) calculated spectrum of pressure fluctuations at $(x, y) = (0, 2.05)$.

Figure 1b shows the calculated density gradient, $\partial\rho/\partial x$, profiles across the flow in the vicinity of the cavity. Both the Freund ABC and PML schemes give the same profiles, which are in excellent agreement with previous schlieren experiments (see Ref. 13) for $M = 0.64$. The spectrum of the pressure fluctuations at the point $(x, y) = (0, 2.05)$ was deduced from the time-stationary pressure signal and is shown in Fig. 1c. The first two basic Strouhal frequencies could be predicted using Rossiter's semiempirical formula,[14]

$$Sr_n = f_n L/U_\infty = (n - \gamma)/(M + 1/\kappa), \qquad n = 1, 2, \ldots, \quad (1)$$

where Sr_n is the Strouhal number corresponding to the nth mode frequency f_n, and $\kappa = 0.25$ and $\gamma = 1/1.75$ are empirical constants corresponding to the average convection speed of the disturbances in the shear layer and the phase delay, respectively. For the $M = 0.6$, the first two modes deduced from Eq. (1) are $Sr_1 = 0.32$ and $Sr_2 = 0.74$. The first two basic Strouhal frequencies determined from the calculations are 0.4 and 0.73, respectively. There is a small error of about 1% in the prediction of Sr_2, but the error for Sr_1 is about 25%. Krishnamurty[13] measures an $Sr_2 = 0.74$, which is very close to the present solution. Therefore, the far-field solution at the buffer region boundaries for cavity flow is well approximated by an incompressible Blasius boundary layer. A comparison of the performance of the nonreflecting boundary schemes is made with the time history of $L_2(p)$ and $L_\infty(p)$ at the physical inlet (boundary I), physical outlet (boundary O), and physical top (boundary T). The results are plotted in Fig. 2 for the range of time $14 < t < 16.5$. It is clear that both the Freund ABC and PML methods could treat boundaries whose velocities are not uniform, such as in boundary layers with

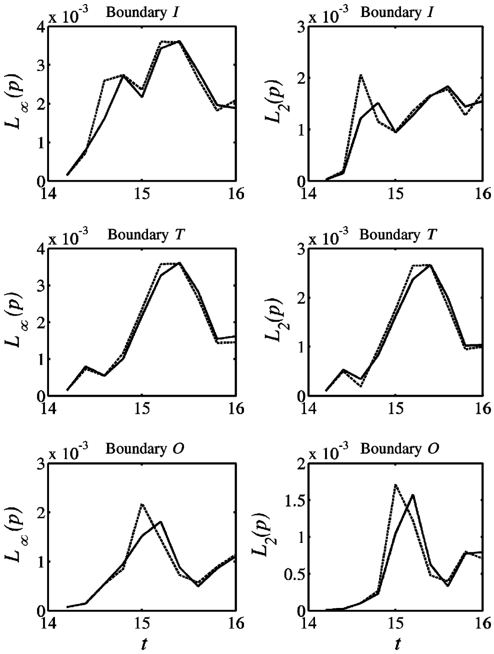

Fig. 2 Time history of error norms L_∞ and L_2 for pressure along the inlet, top, and outlet boundary, boundary conditions: ——, Freund ABC and · · · ·, PML.

high shear flows. In addition, for the same buffer layer width, the errors in the calculated pressure are comparable.

Acoustic Wave Transmission over a Backward-Facing Step in a Duct

In the calculation of a backward-facing step flow in a duct, it is relatively easy to obtain good results for the aerodynamic flowfield. Numerous simulation results have been reported in the literature. Most studies were focused on examining the effects of the Reynolds number and the expansion ratio on the flow pattern and behavior behind the step.[10,15] In this study, focus is placed on the acoustics field, and this requires careful treatment of the boundaries, especially if they are to be truly nonreflecting so that there will be no spurious waves bouncing back to contaminate the aeroacoustics simulation. Note that the magnitudes of the acoustics quantities are only from 10^{-3} to 10^{-5} of the corresponding aerodynamic quantities. Therefore, even a small error in the aerodynamic solution could create spurious acoustics waves that could greatly affect the simulation result. To claim correctness for the acoustics simulation, it is necessary to ascertain that the basic aerodynamic solution is itself correct. Therefore, the first step is to carry out an aerodynamic simulation of an experimental backstep flow and to compare the calculated results with measurements.

The backstep flow investigated by Armaly et al.[16] is shown in Fig. 3. According to their experimental setup, a parabolic velocity profile with mean velocity U_m is specified at the inlet. Here, L_{ref} is taken to be the backstep height H and $U_{\mathrm{ref}} = U_m$. The other flow and geometric conditions are specified as $Re = U_m H/\nu = 400$, $M = 0.1$, and $H_0/H = 1.94$, where H_0 is the height of the duct downstream of the backstep. According to the experimental measurements of Armaly et al.,[16] there are three basic reattachment and separation lengths in this problem. These three lengths are $x_1 = 9.5$, the reattachment length of the recirculation region behind the step; $x_4 = 8.5$, the location of the separation point on the wall opposite the step; and $x_5 = 13.2$, the reattachment length of the separation bubble. A good test of the numerical method would be to compare the prediction of these three lengths. The aerodynamic simulation is carried out using the same parabolic profile for the inlet velocity profile, and the wall boundary conditions are specified as isothermal no-slip conditions. The numerically determined reattachment and separation lengths are $x_1 = 9.6$, $x_4 = 8.1$, and $x_5 = 13.6$, which are in good agreement with measurements.

For the aeroacoustics problem, the acoustics fluctuations are specified as $\hat{\rho} = \rho - \rho_s$, $\hat{u} = u - u_s$, $\hat{v} = v - v_s$, and $\hat{p} = p - p_s$, where

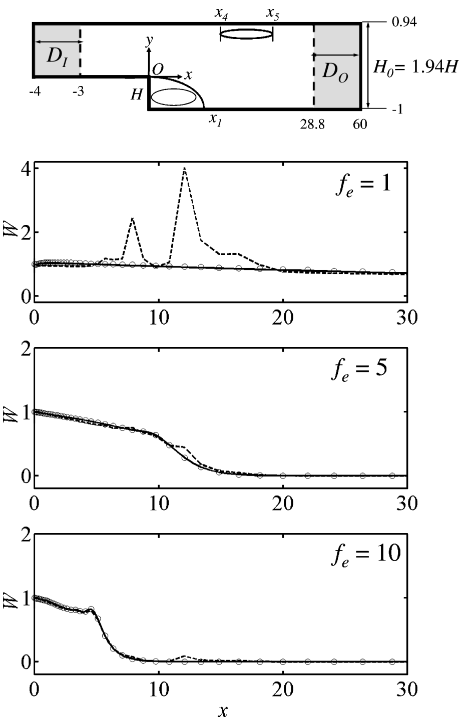

Fig. 3 Distributions of acoustic power W along expansion duct at different f_e: ——, reference solution; - - - -, Freund ABC; and ○, PML.

(ρ_s, u_s, v_s, p_s) are the stationary solutions of the backstep flow that have been determined earlier. Based on this stationary solution, continuous acoustics waves are introduced at the physical inlet, $x = -2$. With the foregoing definitions, three kinds of acoustic waves are examined. Their normalized wavelengths λ_e are 5, 1, and 0.5, respectively. These acoustic waves are introduced into the flow with small pressure disturbances at the physical inlet, so that

$$p = p_s + \tilde{p}\sin(\omega_e t), \qquad \tilde{p} = 6 \times 10^4 p_0$$
$$\omega_e = 2\pi f_e = 2\pi/M\lambda_e \tag{2}$$

In the presence of a mean flow, the instantaneous acoustic intensity is generalized[17] as

$$\boldsymbol{I}(t) = p\hat{\boldsymbol{u}} + (\boldsymbol{M}\cdot\hat{\boldsymbol{u}})(\boldsymbol{M}p + \rho c\hat{\boldsymbol{u}}) + \boldsymbol{M}(p^2/\rho c) \tag{3}$$

where p and $\hat{\boldsymbol{u}}$ are the acoustic quantities and others are mean flow quantities. For a general cross section at constant x, the acoustic power W passing through this section can be defined as the integral of the acoustic intensity

$$W = \frac{1}{\boldsymbol{T}}\int_0^T \int_{-1}^{0.94} \boldsymbol{I}_x \,\mathrm{d}y\,\mathrm{d}t \tag{4}$$

Therefore, a good test of the validity and extent of the nonreflecting schemes would be to compare their performance in the calculation of W. Theoretically, the backstep flow is like a low-pass filter to the acoustic waves. Low-frequency waves could pass and propagate for a long distance downstream. High-frequency waves are strongly attenuated. Figure 3 shows three acoustic power distributions along the duct for $f_e = 1$, 5, and 10. This comparison clearly shows that PML is best for all three frequencies tested, low to high. The Freund ABC can predict the power fairly correctly for moderate to high frequencies. Even then, the error shown for the $f_e = 5$ case is quite noticeable. As f_e is reduced to 1, the error compared to the reference solution is at least one order of magnitude larger than that given by the PML.

Conclusions

This study reports on a comparison of different nonreflecting boundary schemes for one-step simulation of duct aeroacoustics. A

DNS scheme based on a finite difference method is used to solve the governing equations. Two kinds of nonreflecting boundary schemes are compared: the Freund ABC[6] and the PML by Hu.[7] Two cases are calculated: 1) flow over a cavity and 2) transmission of acoustic waves over a backstep flow. For each case, a benchmark reference is calculated using a large computational domain. The Freund ABC and the PML give essentially the same error for the cavity case with an identical computational region. On the other hand, the Freund ABC gives rise to solutions with increasing error as f_e decreases for the backstep flow case. The error increases to approximately one order greater than that given by the PML at $f_e = 1$ with the same buffer region. Furthermore, for the same level of errors, the PML requires a smaller computational domain, has a wider frequency range, and requires less computational time for the same duct aeroacoustics simulation problem.

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